Long-Short Portfolio

Leo Schubert[1]
Constance University of Applied Sciences

Abstract

Long-short strategies are one of the most successful tools, applied by hedge funds manager. One under-evaluated stock is bought (long position) and an over-evaluated stock is sold (short position) at the same time. After a short term, when the values of the stocks are as expected, profit can be realized by a closing transaction. The possibility to find first obvious over- and under-evaluated stocks depends on the number of participants in this markets. While the hedge funds strategies become more popular, the chance to achieve profit by this strategies is shrinking.

Therefore two models to generate long-short portfolios are proposed. By this approaches a portfolio A for the long- and a portfolio B for the short position were computed. The difference of the values of A and B is designed to oscillate from negative to positive and reverse. This behavior of oscillating or mean reverting stock prices was stated by e.g. E. Fama and K. R. French (1988). Mean reversion of portfolios can offer the possibility of statistical arbitrage. The proposed linear models were tested by stocks of the Tokyo stock exchange. The results seem to be applicable and show an additional advantage of low systematic risk.

Key-Words: Portfolio management, hedge funds, long-short-strategy, mixed integer linear optimization, statistical arbitrage, single-index-model, beta-factor, mean reversion

1. Introduction

Besides active and passive portfolio management, in recent years portfolios were constructed, which produce profit in bearish markets, too. Examples are the so called Hedge-Funds[2]. This funds claim to be hedged, because of e.g. the long-short-strategies they use to be market independent. By this strategy, investors can earn in bullish and in bearish markets.[3] To apply this strategy means e.g. to buy an asset with underestimated value and to sell an asset with overestimated value at the same time. If the expectations concerning the mean of the both assets are correct, the price of the both assets will move back to the mean in some weeks or months. Then, the closing transaction offers an market independent return. The long-short-strategy can also be applied, if two assets A and B seem to have equal mean value. In this case, the market beliefs, that the value of the both is comparable. The difference between the price of A and B is temporary and offers another application of a long-short-strategy.

Compared with traditional portfolio management[4], long-short-strategies do not interpret the expected return μ itself but e.g. the temporary deviation of this μ as valuable chance. Therefore a long-short-position will be closed after a short time interval when the deviation is disappeared. There is no long-term investment planned.

The knowledge of the mean reverting behavior of stock prices offers a kind of statistical arbitrage. Like in the case of the arbitrage, to achieve profit, it is important to be the first, who recognizes the deviation of the equilibrium or of the mean. Due to the increasing number of

[1] schubert@htwg-konstanz.de
[2] Rutkis A. (2002) gives a view of hedge funds strategies.
[3] If the market is either bullish nor bearish, this strategy seem to be less successful.
[4] see Markowitz H. (1952); Sharp W. F. (1964)

participants in the markets, searching for arbitrage, the possibility to apply successful a long-short-strategy becomes smaller.

Beside taxation often reduces the profit of investors. To estimate the investors portfolio selection behavior, the system of taxation must be respected. The after tax profit of capital gains is dependent on, whether capital gains are taxed and losses are deduced without restriction at the same rate or not or whether capital gains are taxed only if realized within 12 month like in Germany etc.. Taxation can produce a wide range of distortion in portfolio behavior. The proposed approaches below occur in absence of non-neutral distortioning taxation[5].

In the capital market theory the above descried behavior of stock prices is known as "Mean Reversion[6]". Whether the mean reverting process stays or not, when portfolios instead of single stocks are regarded, is unknown. If mean reversion exists, it must be observable and util for statistical arbitrage when the mean reverting behavior remains in the near future.

To investigate mean reversion of portfolios, two models where designed. Instead of two stocks like in Figure 1 for the long- and the short position, two portfolios A and B must be found by the models. The difference of the values of A and B should be oscillating from negative to positive and reverse. By this, the mean of the difference of the two portfolios can be expected as about zero. The value of the portfolio (+A-B) should revert to its mean of zero in a fixed time interval. The two models will be called "Max Tau" and "Max Sum" in the following. After the introduction of the models, they will be tested by empirical data of the Japanese stock exchange.

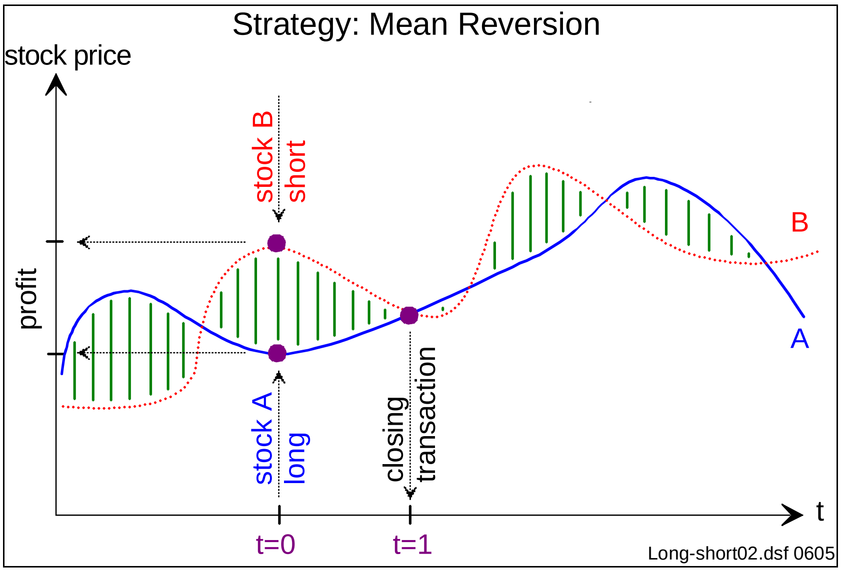

Fig. 1: Mean Reversion

2. Long-short portfolio: Max Tau

Portfolios A and B with the behavior shown by the two stocks in Figure 1, can be designed by a linear mixed integer model. For this purpose the time t=1, ..., T was divided into subsets T_k (k=1, .., m) which may not be exhaustive.

The values v_{it} of the i=1, ..., n assets at time t=1, ..., T are used directly as parameters. The budget C, divided by the value v_{iT} of asset i at the time T shows the number of assets x_i

[5] A wide range of tax-induced distortions are discussed in Auerbach, A. J., Hines Jr., J. R. (2002) and Poterba, J. M. (2002).

[6] see Fama E., French K. R. (1988); Poterba J. M., Summers L. H. (1988)

(i=1, ..., n) which can be bought at T. The product $x_i\, v_{it}$ is the amount invested in asset i. The sum of this amount is the value of the long-short portfolio (+A-B) or the distance d_t (t= 1, ..., n) between the values of A and B (see inequalities (2a)). The budget C in the model is equal for every asset but should be individual fixed for every asset dependent on the possibilities to buy or sell this asset. Portfolio A will contain stocks with $x_i > 0$ and portfolio B stocks with $x_i < 0$.

If the distances d_t ($t \in T_k$) between the values of the two sub-portfolios should be positive, the distances d_t ($t \in T_{k+1}$) should be negative and reverse. In this subsets, the distance level τ resp. $-\tau$ should be at least once met. This distance level τ will be maximized by the objective function (see (1)).

In equality (2b) the variables c_i^- and c_i^+ (i=1, ..., n) measure as slack-variables what amount of asset i should be sold (c_i^-) or bought (c_i^+). The sum of both is in inequality (2c) restricted at least to the twice of the budget C. This means, that most of the long positions are financed by short-selling or that the sum of the investments is closed to zero in T.

To get high deviations between the values of the two sub-portfolios, the objective function of the model must

$$\text{maximize } \tau \tag{1}$$

under the conditions

$$\sum_{i=1}^{n} x_i v_{it} = d_t \,; \quad \text{with} \quad -C/v_{iT} \le x_i \le C/v_{iT}, \ (i = 1, ..., n) \tag{2a}$$

$$v_{iT}\, x_i + c_i^- - c_i^+ = 0, \quad (i = 1, ..., n) \tag{2b}$$

$$\sum_{i=1}^{n} c_i^- + c_i^+ \le 2C \tag{2c}$$

and

$$d_t \ge (\delta_t^o - 1)\, M + \tau\,, \quad (t \in T_k, \ \ k = 1, 3, 5, ..., m-1) \tag{3a}$$

$$d_t \le \tau - \varepsilon + \delta_t^o\, M, \quad (t \in T_k, \ \ k = 1, 3, 5, ..., m-1) \tag{3b}$$

$$\sum_{t \in T_k} \delta_t^o \ge 1\,, \quad (k= 1, 3, 5, ..., m-1) \tag{3c}$$

$$d_t \le (1 - \delta_t^u)\, M + \tau\,, \quad (t \in T_k, \ \ k = 2, 4, 6, ..., m) \tag{4a}$$

$$d_t \ge \tau + \varepsilon - \delta_t^u\, M, \quad (t \in T_k, \ \ k = 2, 4, 6, ..., m) \tag{4b}$$

$$\sum_{t \in T_k} \delta_t^u \ge 1\,, \quad (k= 2, 4, 6, ..., m) \tag{4c}$$

with ε: small number, M: big number.

The restrictions (3a) - (3c) force the model, to produce at least once a distance $d_t \ge \tau$ ($t \in T_k$) in subset T_k. The restrictions (4a) - (4c) guaranties, that the solution has in the following subset T_{k+1} also at least once a distance $d_t \le -\tau$ ($t \in T_{k+1}$). If the distance τ resp. $-\tau$ is met, the binary variables δ_t^o resp. δ_t^u in (3) resp. (4) become equal one. The unequality (3c) resp. (4c) counts this cases. Within two subset $T_k \cup T_{k+1}$ the distance d_t of the two portfolios A and B must be about zero at least once (see Figure 2 – left side).

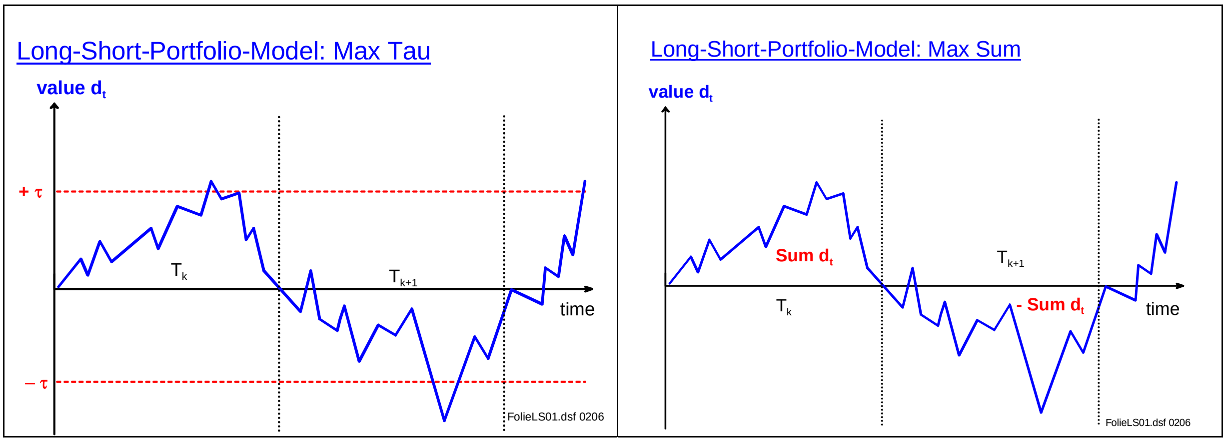

Fig. 2: Long-Short-Portfolio-Models

To reduce the management costs of the portfolio, additional restrictions to the number of stocks can be integrated:

$$x_i - M\,\delta_i^+ \le 0 \quad (i=1, ..., n) \quad \text{and} \quad \sum_{i=1}^{n} \delta_i^+ \le z^+ \tag{5a}$$

$$x_i + M\,\delta_i^- \ge 0 \quad (i=1, ..., n) \quad \text{and} \quad \sum_{i=1}^{n} \delta_i^- \le z^- \tag{5b}$$

with

- δ_i^+, δ_i^-: binary variables
- z^+, z^-: max. number of assets long (z^+) or short (z^-)
- M: big number.

For an empirical test of the "Max Tau" long-short model, data from the Japanese capital market were used. The data base were the 86 biggest Japanese stocks which were listed in the stock exchange in Tokyo throughout the period from September 5th 1988 until November 1st 1999 (some selected stocks are listed in Table 1). For the optimization, only the daily stock values of this stocks from January 3rd 1994 until December 31th 1998 were used. The total number of values v_{it} for each of the 86 stocks were 1304. In the following 10 months (until the November 1st 1999) after the optimization or the following 217 values v_{it} were used to control the behavior of the optimized portfolio. If the models produce portfolios A and B which revert to their mean value, this behavior must be shown in this 10 month, too.

For the optimization the time was divided into 5 subsets T_k each with about 261 values v_{it}. One subset corresponds with one year. To determine the positive and negative bounds of the variables $x_1, ..., x_n$, an uniform max. budget C = 0.5 Mio Yen was used for every asset in (2a). The absolute Budget of long- and short positions is $2C \le 1$ Mio Yen. Figure 3a illustrates the solution of an example in which the number of assets in the portfolios A resp. B was not constrained by inequalities (5). The realization of such portfolios can fails, due to the rare shortselling possibilities in financial markets. Therefore in Figure 3b the number of assets which should be sold was restricted to $z^- = 1$ (see (5b)). The difference d_t between the Portfolio A and B was in the example of Figure 3a at least $\tau = 0.161$ Mio Yen in each subset T_k and in the example of Figure 3b at least $\tau = 0.123$ Mio Yen. The vertical lines divide the 5 subsets T_k. Each subset contains about 260 days resp. values v_{it}. The smaller subset at the right side is related to the 10 month after the 31.12.1998. The optimization process was in both cases aborted after some hours of consumed CPU-time.

The unrestricted portfolio contains 10 assets long and 9 assets short and the restricted portfolio 3 resp. 1 asset. The blue chart is the value of the portfolio A (long position) and the

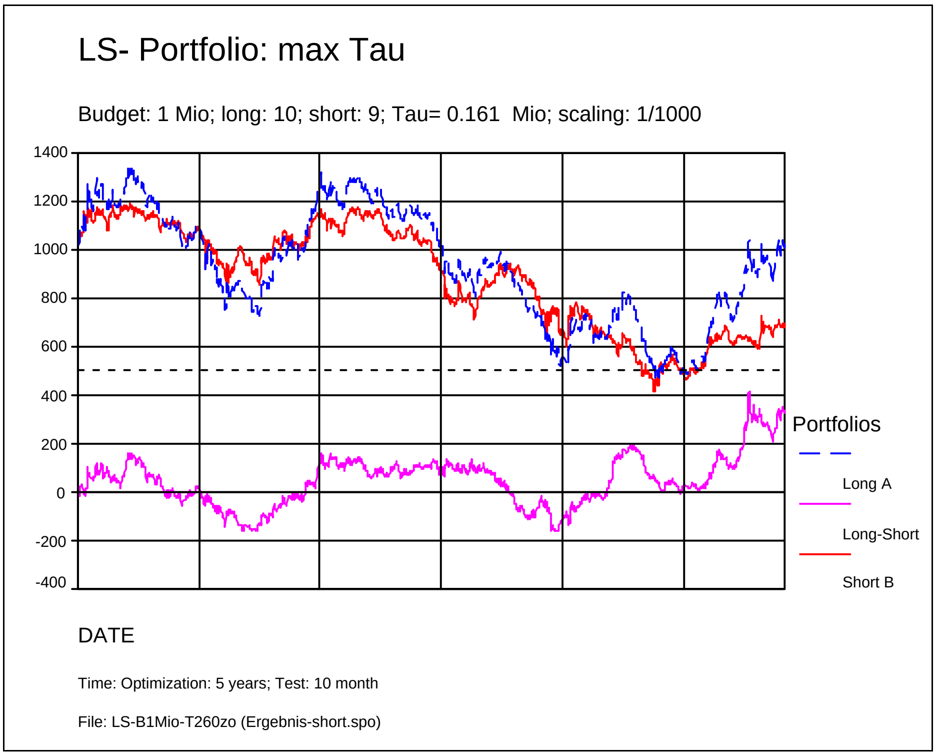

Fig. 3a: Model “Max Tau” without restrictions

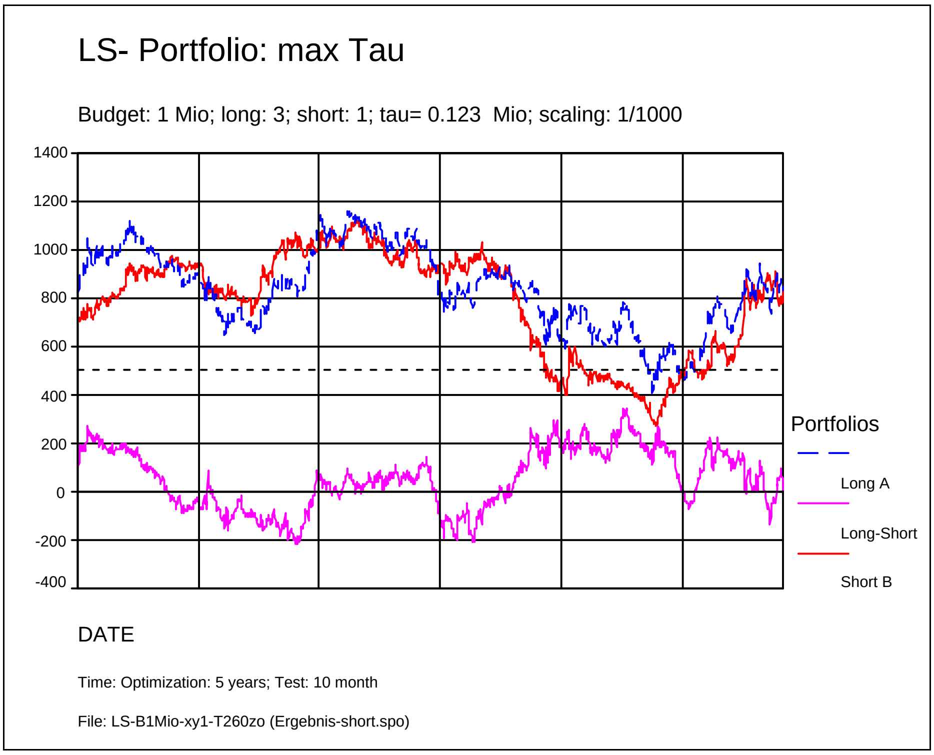

Fig. 3b: Model “Max Tau” with one asset sold

red chart of the portfolio B (short position). At T = 1.1.1999 the value of both portfolios is 0.5 Mio (see horizontal dotted line). After this date, the distance of the value of the two portfolios is growing in Figure 3a. In Figure 3b this distance shows in the test-interval a similar behavior like in the optimization periods. The chart of the long-short portfolio crosses the zero-line and offers a possibility for statistical arbitrage.

Within the optimized time interval the solutions of the model “Max Tau” offer portfolios with the expected behavior (see Figure 2). The CPU-time for finding an acceptable good solution (without knowing whether it is the optimal solution) was high, due to the amount of binary variables and structures used. Therefore, a second algorithm “Max Sum” without binary variables was tested, although the objective function is not searching high extreme values of the distance d_t in every subset T_k.

3. Long-short portfolio: Max Sum

The following approach to find a long-short portfolio is linear. Like in the model “Max Tau”, subsets of the time are used. For each subset T_k (k=1, .., m) the sum of distances d_t ($t \in T_k$) is computed. This sum should be positive in the first subset, negative in the second subset and positive in the following and so on (see Figure 2 - right side). Like in the model “Max Tau”, the sum of distances d_t should be positive with a value of at least +Sum in the first subset and negative and a value of at most –Sum in the second subset and so on.

To achieve portfolios A and B the objective function

$$\text{maximize Sum} \tag{6}$$

under the conditions (2a)-2(c) and

$$\sum_{t \in Tk} d_t \geq +\text{Sum}, \qquad (k = 1, 3, 5, ..., m-1) \tag{7a}$$

$$\sum_{t \in Tk} d_t \leq -\text{Sum}, \qquad (k = 2, 4, 6, ..., m). \tag{7b}$$

To reduce the management costs of the portfolio, restrictions like above can be integrated (see (5)).

For an empirical test of the model “Max Sum” data from the Japanese capital market were used like in the empirical test of the model “Max Tau”.

In Figure 4a the solution without restriction is shown and in Figure 4b the number of short positions was restricted to one. The shape of the chart of the long-short portfolio in Figure 4a is similar to the shape of the chart in Figure 3a. This is surprising, because of the different stocks the two long-short portfolios contain (see Table 1). In the case of Figure 3b and 4b, the portfolio B is identical. It contains the stock MITSUBISHI ELECTRIC CORP. Therefore, the shapes of the long-short portfolios are similar. The CPU-time for solving the two examples was below one second.

Like above, the restricted long-short portfolio (Figure 4b) gives a better forecast of the behavior of the long-short portfolio compared with the unrestricted case. The value of the objective function is an aggregated value (Sum = 37.5 Mio resp. 22.4 Mio). To compare this value with τ, it is better to use the average value, although the average value will be smaller than the maximum value τ. If each subset T_k contains 260 time intervals, the average distance d_t would be at least 0.144 Mio resp. 0.086 Mio Yen. Compared with the two τ values of the examples above, the solution of the “Max Sum” model seem to produce comparable results.

To see the influence of the size of the subsets T_k, only 130 time intervals where used in the example of Figure 4c. Now, the chart of the long-short portfolio has a rapid changing direction. In every T_k several positive and negative extremes can be observed. The deviations are within the interval +/- 0.1 Mio (see red horizontal lines). The subsets T_k seem to be too small selected.

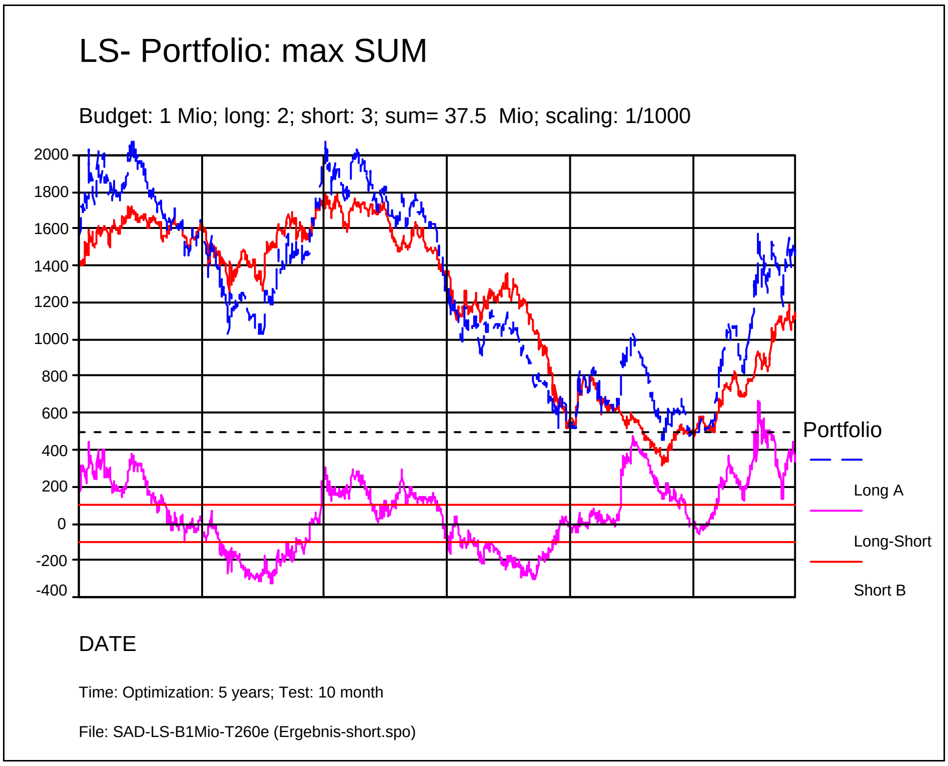

Fig. 4a: Model “Max Sum” without restrictions

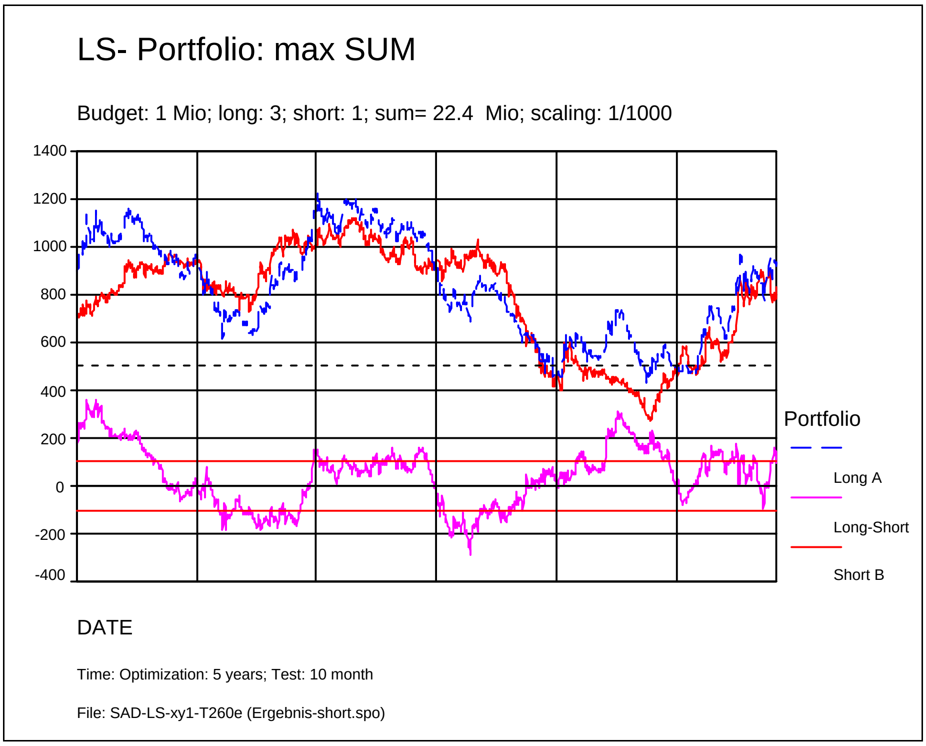

Figure 4b: Model “Max Sum” with one asset sold

4. Mean-variance- and long-short portfolios

The above designed long-short portfolios do not have a long term strategy and therefore the mean-variance space is not the right frame for this instruments. Nevertheless it seems to be interesting, to regard the traditional measurements for risk like the variance or the β.

No. Stock	Tau-T260	SUM-T260	No.	Tau-T260-ls1	SUM-T260-ls1
03. ALL NIPPON AIRWAYS Y50	**-0,1870**	0,0000	X03	0,0000	0,0000
11. CASIO COMPUTER CO	**0,0421**	0,0000	X11	0,0000	0,0000
12. CITIZEN WATCH CO	**0,1005**	0,0000	X12	0,0000	0,0000
15. DAIWA SECURITIES GROUP	**0,1947**	0,0000	X15	0,0000	0,0000
18. FUJI BANK	0,0000	**-0,2679**	X18	0,0000	0,0000
23. INDUSTRIAL BANK OF JAPAN	**-0,1402**	**-0,1446**	X23	0,0000	0,0000
25. JAPAN AIRCRAFT MFG CO	**-0,0253**	0,0000	X25	0,0000	0,0000
26. JUSCO CO	**0,1899**	0,0000	X26	0,0000	**0,2227**
29. KAWASAKI STEEL CORP	**0,0503**	0,0000	X29	0,0000	0,0000
33. MARUI CO	0,0000	0,0000	X33	0,0000	**0,3626**
36. MINOLTA Y50	**0,0576**	**0,0640**	X36	0,0000	0,0000
38. MITSUBISHI ELECTRIC CORP	0,0000	**-0,5875**	X38	**-1,0000**	**-1,0000**
39. MITSUBISHI ESTATE CO	**-0,1663**	0,0000	X39	**0,6831**	0,0000
41. MITSUBISHI TRUST	**-0,0525**	0,0000	X41	0,0000	0,0000
46. NIKKO SECURITIES Y50	**0,1947**	**0,9360**	X46	**0,3102**	**0,4147**
50. NIPPON TELEVISION NETWORK	**0,0652**	0,0000	X50	0,0000	0,0000
53. OJI PAPER CO	**-0,0481**	0,0000	X53	0,0000	0,0000
58. SAKURA BANK	0,0000	0,0000	X58	**0,0067**	0,0000
60. SANWA BANK	**-0,1167**	0,0000	X60	0,0000	0,0000
67. SHISEIDO CO	**-0,1947**	0,0000	X67	0,0000	0,0000
71. SUMITOMO CORP	**0,0179**	0,0000	X71	0,0000	0,0000
72. SUMITOMO ELECTRIC IND	**-0,0163**	0,0000	X72	0,0000	0,0000
76. TOKAI BANK	**0,0871**	0,0000	X76	0,0000	0,0000

Table 1: Stocks, selected by the portfolios

The mean-variance space of Figure 5 contains some efficient portfolios, an index portfolio (with equal weighted stocks), the 86 stocks, a long-short portfolio with maximized α (while β=0) and the above computed long-short portfolios (see data in Table 2). The return and the standard deviation refer to returns per day.

Base for long-short portfolio with maximized α (while β=0) is the single index model of W. F. Sharpe[7]. Such portfolios must not be designed heuristically,[8] they can be constructed by linear optimization. This portfolio will produce market independent high returns if the α of the portfolio is high. Therefore the assets with high positive α_i must be bought and the assets with negative α_i must be sold to get a portfolio with high market independent returns.

The unrestricted long-short portfolios discussed above have low variance and low market risk with $\beta \leq 0.1$ (see Table 2). With the condition that only one stock should be sold, the systematic risk is about $\beta = +/- 0.3$. For this cases, the standard deviation of the returns is higher too (see Figure 5 and Table 2). The highest return and risk measured by the standard deviation has the portfolio with maximized α. The risk measure β for the systematic risk is minimal (β=0). It is surprising, that portfolios without restrictions on the number of stocks and low market risk (β< 0.1) have their position closed to the index with a systematic risk of β = 1.

[7] see Sharpe W. F. (1964).
[8] see Farrell, J. L. (1997), pp. 259ff.

The return of the long-short portfolios of the model "Max Tau" and "Max Sum" is closed to zero, due to their construction which forces the portfolios to produce positive and negative returns. For statistical arbitrage, the times of negative returns can also be used to produce profit. Therefore, the absolute value of the returns would be a more adequate measure of return of this long-short portfolios.

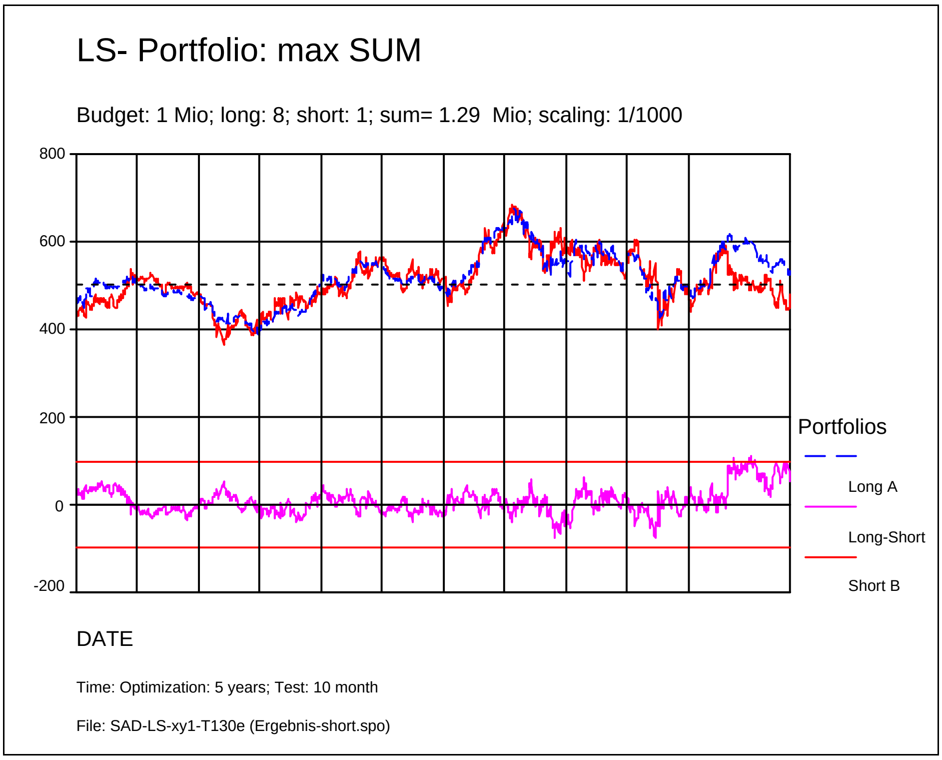

Fig. 4c: Model "Max Sum" with one asset sold and subset with 130 days

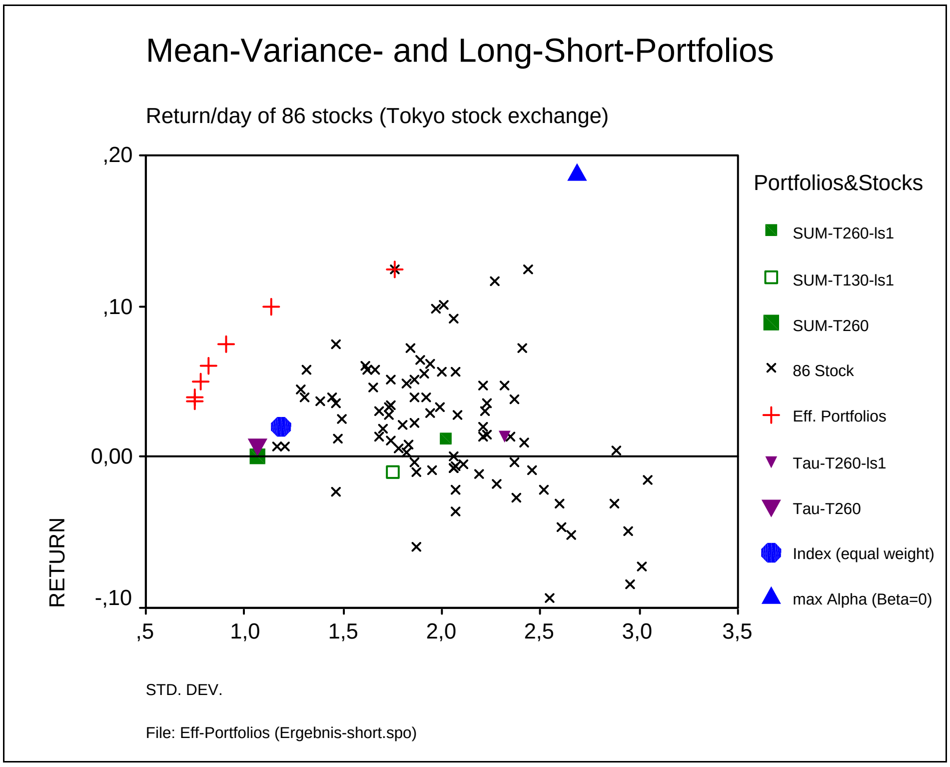

Fig. 5: Mean-Variance- and Long-Short-Portfolios

Portfolio-case	Fig.	Return	Std. Dev.	Alpha	Beta	No. Stocks Long	No.Stocks Short
Index (equal weighted)		0,0206	1,1813	0,0000	1,0000	1	2
Eff. Portfolio-MVP		0,0369	0,7462	0,0261	0,5231	17	0
Eff. Portfolio-0.04		0,0400	0,7485	0,0292	0,5235	16	0
Eff. Portfolio-0.05		0,0500	0,7733	0,0389	0,5414	16	0
Eff. Portfolio-0.06		0,0600	0,8158	0,0482	0,5726	20	0
Eff. Portfolio-0.075		0,0750	0,9088	0,0622	0,6235	17	0
Eff. Portfolio-0.10		0,1000	1,1369	0,0853	0,7133	10	0
Eff. Portfolio-0.12		0,1246	1,7636	0,1078	0,8178	1	0
max Alpha (Beta=0)		0,1868	2,6827	0,1868	0,0000	1	2
TAU-T260	3a	0,0083	1,0615	0,0061	0,1099	10	9
TAU-T260-ls1	3b	0,0145	2,3199	0,0065	0,3889	3	1
SUM-T260	4a	0,0004	1,0711	-0,0003	0,0340	1	3
SUM-T260-ls1	4b	0,0121	2,0202	0,0100	0,1026	3	1
SUM-T130-ls1	4c	-0,0097	1,7522	-0,0025	-0,3540	8	1

Table 2: Measures of risk and return

5. Conclusions

Although the test of the long-short portfolios is based on few examples some results and questions were obvious. Statistical arbitrage is not without risk like arbitrage. The desired behavior of the value of the long-short portfolio can be well produced for the optimization time interval. Important to achieve profit is, that the observed or expected relationship or behavior of the two portfolios will also be true for the time of investment. For this interval, the risk to change the behavior can not be reduced to zero. The proposed models do not always produce portfolios with the same mean reverting behavior in the post optimization time. The results show, that mean reversion behavior sometimes seem to be valid for the prices of portfolios, too. Nevertheless some economic reasons based on the economic sectors of the stocks and sub-portfolios etc. would be important for creating trust in this long-short strategies. The models can help to discover such relationships. Compared with traditional portfolio risk measure, some of the long-short portfolios show low standard deviation as well as low β resp. systematic risk.

The branch & bound algorithm CPLEX used for solving the "Max Tau" model was always aborted. The solutions found at this moment were nevertheless interesting. The results of the two models are very similar. If the CPU-time for solving the "Max-Tau" model can not be reduced, the model "Max Sum" seem to be a very good alternative model. This model has the feature to select less stock for the portfolio than the "Max Tau" model or the traditional models which search efficient portfolios.

6. References

1. Auerbach, A. J., Hines Jr., J. R. (2002): Taxation and Economic Efficiency, in Auerbach, A. J., Feldstein, M.: Handbook of Public Economics, vol. 3, pp. 1109-1171.
2. Fama, E.; French, K. R. (1988): Permanent and Temporary Components of Stock Prices, The Journal of Political Economy, Vol. 96, No. 2, pp. 246-273.
3. Farrell, J. L. (1997): Portfolio-Management, Mc Graw Hill, New York.
4. John, O. W. (2000): Anlagestrategie: Rendite ohne Marktrisiko, Die Bank, 11/2000, p. 768-770.
5. Markowitz, H. (1952): Portfolio Selection , Journal of Finance, Vol. 7, p. 77-91.
6. Poterba, J. M. (2002): Taxation, Risk-Taking, and the Household Portfolio Behavior, in Auerbach, A. J., Feldstein, M.: Handbook of Public Economics, vol. 3, pp. 1109-1171.

7. Poterba, J. M., Summers, L. H. (1987): Mean Reversion in Stock Prices: Evidence and Implications, National Bureau of Economic Research Inc., working paper No. 2343.
8. Rutkis, A. (2002): Hedge-Fonds als Alternative Investments – Stilrichtungen, Risiken, Performance, Bankakademie Verlag GmbH.
9. Sharpe, W. F., (1964), Capital Asset Prices: A Theory of markets equilibriums under conditions of risk, Journal of Finance, Nr. 19, p. 425-442.